

# Meson-Baryon Couplings and the F/D ratio in Light Cone QCD

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## Abstract

Using the general form of the baryon currents, we calculate the meson-baryon coupling constants and the F/D ratio within the framework of light cone QCD sum rules in the  $SU(3)$  flavor symmetry limit. The dependence of the results on the Dirac structure and on the free parameter  $b$  appearing in the general baryon current is considered. Comparison of our results on F/D ratio with the existing results is presented.

# 1 Introduction

Determination of the various fundamental parameters of hadron from experimental data requires information about physics at large distance. Unfortunately such information can not be achieved from the first principles of a fundamental theory of strong interactions due to its very complicated infrared behavior. For this reason to determine properties of hadrons a reliable non-perturbative approach is needed. Among all non-perturbative approaches, QCD sum rules [1] are especially powerful in investigating the properties of low lying hadrons. Note that in traditional sum rules non-perturbative effects are taken into account through various condensates. Among various applications, determination of meson baryon couplings is of particular interest, since they are main ingredients of baryon-baryon interactions. Recently, the two point correlation function of the nucleon with a pion

$$\Pi = i \int d^4x e^{ipx} \langle \pi(q) | \mathcal{T} \eta_N(x) \bar{\eta}_N(0) | 0 \rangle \quad (1)$$

has been extensively used to calculate pion-nucleon coupling in the framework of QCD sum rules [2-9].

This correlator function contains three different Dirac structures: a)  $i\gamma_5$ ; b)  $\gamma_5\sigma_{\mu\nu}q^\mu p^\nu$ ; and c)  $i\gamma_5 \not{p}$ , each of which can in principle be used to calculate the meson-baryon coupling. In [4], it is shown that the predicted pion-nucleon coupling depend on the Dirac structure. In [5, 6] sum rules for the  $i\gamma_5$  structure and in [7] for  $\gamma_5\sigma_{\mu\nu}q^\mu p^\nu$  structure beyond the chiral limit were obtained. Both sum rules yield the  $\pi NN$  coupling close to its experimental value, while the  $i\gamma_5 \not{p}$  sum rules contain large contributions from the continuum and for this reason its predictions are not reliable. The pseudoscalar and tensor sum rules beyond the chiral limit have been applied to other meson-baryon couplings  $\eta NN$ ,  $\pi\Xi\Xi$ ,  $\pi\Sigma\Sigma$ , and  $\eta\Sigma\Sigma$  [6, 7]. It is well known that the meson-baryon couplings in  $SU(3)$  limit can be classified in terms of two parameters: the  $\pi NN$  coupling and the so called F/D ratio [10]. The sum rules for the F/D ratio for general form of baryon currents for above mentioned three different Dirac structures in the framework of traditional sum rules is studied in [11].

In this work, our aim is to study F/D ratio in the framework of an alternative approach to the traditional sum rules, namely light cone QCD sum rules(LCQSR) and compare the predictions of these different approaches. LCQSR is based on the operator product expansion on the light cone, which

is an expansion over the twists of the operators rather than the dimensions in the traditional sum rules and the main contribution comes from the lowest twist operators. The matrix elements of the nonlocal operators sandwiched between a hadronic state and the vacuum defines the hadronic wave functions (about the light cone QCD sum rules see [12, 13] and references therein).

The paper is organized as follows. In Sect. II, the light cone sum rules for meson baryon coupling using general baryon currents for the structures  $i\gamma_5 \not{q}$  and  $\sigma_{\mu\nu}\gamma_5 p^\mu q^\nu$  are obtained. More over, we construct an expression for the F/D ratio from OPE. Sect. III contains our numerical results and conclusions.

## 2 Sum Rules for the Meson-Baryon Couplings and the F/D Ratio

As we have noted, meson-baryon couplings in the  $SU(3)$  flavor symmetry limit can be expressed in terms of only two parameters [10]  $g_{\pi NN}$  and  $\alpha = \frac{F}{F+D}$  as:

$$\begin{aligned} g_{\eta NN} &= \frac{1}{\sqrt{3}}(4\alpha - 1)g_{\pi NN}, \quad ; \quad g_{\pi \Xi \Xi} = (2\alpha - 1)g_{\pi NN}, \\ g_{\eta \Xi \Xi} &= -\frac{1}{\sqrt{3}}(1 + 2\alpha)g_{\pi NN}, \quad ; \quad g_{\pi \Sigma \Sigma} = 2\alpha g_{\pi NN}, \\ g_{\eta \Sigma \Sigma} &= \frac{2}{\sqrt{3}}(1 - \alpha)g_{\pi NN}, \end{aligned} \quad (2)$$

In this section, we will derive light cone sum rules for the  $\pi^0 NN$  coupling. A sum rule for the meson-baryon couplings can be constructed by equating two different representations of a suitably chosen correlator, written in terms of hadrons and quark-gluons. We begin our calculations by considering the following correlator:

$$\Pi = i \int dx e^{ipx} \langle \mathcal{M}(q) | \mathcal{T} J^B(x) \bar{J}^B(0) | 0 \rangle, \quad (3)$$

where  $J^B$  is the current of the baryon under consideration,  $\mathcal{T}$  is the time ordering operator,  $q$  is the momentum of the meson  $\mathcal{M}$ . This correlator can be calculated on one side phenomenologically, in terms of the hadron parameters, and on the other side by the operator product expansion (OPE)

in the deep Euclidean region,  $p^2 \rightarrow -\infty$ , using QCD degrees of freedom. By equating both expressions, we construct the corresponding sum rules.

Saturating the correlator, Eq. (3), by ground state baryons we get:

$$\Pi(p_1^2, p_2^2) = \frac{\langle 0 | J^B | B_1(p_1) \rangle}{p_1^2 - M_1^2} \langle B_1(p_1) \mathcal{M}(p) | B_2(p_2) \rangle \frac{\langle B_2(p_2) | J^B | 0 \rangle}{p_2^2 - M_2^2}, \quad (4)$$

where  $p_2 = p_1 + q$  and  $M_i$  is the mass of the baryon  $B_i$ .

The matrix elements of the interpolating currents between the ground state and the state containing a single baryon,  $B$ , with momentum  $p$  and having spin  $s$  is defined as:

$$\langle 0 | J^B | B(p, s) \rangle = \lambda_B u(p, s), \quad (5)$$

where  $\lambda_B$  is the residue, and  $u$  is the Dirac spinor for the baryon. In order to write down the phenomenological part of the sum rules from Eq. (4) it follows that one also needs an expression for the matrix element  $\langle B(p_1) \mathcal{M} | B(p_2) \rangle$ . This matrix element is defined as:

$$\langle B(p_1) \mathcal{M} | B(p_2) \rangle = g_{\mathcal{M}BB} \bar{u}(p_1) i\gamma_5 u(p_2) \quad (6)$$

With these definitions, the phenomenological representation of the correlator becomes:

$$\Pi_{\mathcal{M}BB}(p_1, p_2) = -\frac{\lambda_B^2 g_{\mathcal{M}BB}}{(p_1^2 - m_B^2)(p_2^2 - m_B^2)} \left( p_1^\mu q^\nu \sigma_{\mu\nu} \gamma_5 - m_B i\gamma_5 \not{q} + \frac{m_\pi^2}{2} i\gamma_5 \right) + \dots \quad (7)$$

where  $\dots$  stands for the contribution of higher states and the continuum. Note that in this work, we will consider massless pion in which case, the pseudo scalar structure  $i\gamma_5$  vanishes identically and hence we will omit this structure. In this work we will both of the remaining structures appearing in Eq. (7) and compare the reliability of the structures.

On the QCD side, in order to evaluate the correlator, one needs suitable expressions for the baryon currents. In this work, we will use the following general forms of baryon currents:

$$J_p(x, t) = 2\epsilon_{abc} \left\{ \left[ u_a^T(x) C d_b(x) \right] \gamma_5 u_c(x) + b \left[ u_a^T(x) C \gamma_5 d_b(x) \right] u_c(x) \right\}, \quad (8)$$

$$J_\Xi(x, t) = -2\epsilon_{abc} \left\{ \left[ s_a^T(x) C u_b(x) \right] \gamma_5 s_c(x) + b \left[ s_a^T(x) C \gamma_5 u_b(x) \right] s_c(x) \right\} \quad (9)$$

$$J_\Sigma(x, t) = 2\epsilon_{abc} \left\{ \left[ u_a^T(x) C s_b(x) \right] \gamma_5 u_c(x) + b \left[ u_a^T(x) C \gamma_5 s_b(x) \right] u_c(x) \right\} \quad (10)$$

where  $C$  is the charge conjugation operator and  $b$  is an arbitrary real parameter. The Ioffe current corresponds to the choice  $b = -1$ . The currents for  $\Xi$  and  $\Sigma$  can be obtained from the nucleon interpolating current via the  $SU(3)$  rotation. Namely  $\Xi$  and  $\Sigma$  currents can be easily obtained from the nucleon current by the following simple replacements: to obtain the  $\Xi$  current, substitute  $s$  and  $u$  in place of  $u$  and  $d$  respectively and to obtain the  $\Sigma$  current, substitute  $s$  in place of  $d$  in the proton current. Note that the pion-nucleon coupling constant for the Ioffe current in LCQSR have been calculated in [17].

In the large Euclidean momentum  $-p^2 \rightarrow \infty$  region, the correlator can be calculated using the OPE. For the pion-proton sum rule, the OPE yields:

$$\begin{aligned}
\Pi_{\pi NN}(p_1, p_2) = & i\epsilon_{abc}\epsilon_{def} \int d^4x e^{ipx} \langle \pi | \bar{u}^d A_i u^a \{ \gamma_5 A_i \gamma_5 \text{Tr} S_d^{be} S_u^{cf} + \\
& + \gamma_5 A_i S_d'^{be} S_u^{cf} \gamma_5 + b^2 A_i \gamma_5 S_d'^{be} \gamma_5 S_u^{cf} + \\
& + b^2 A_i \text{Tr} S_u^{cf} \gamma_5 S_d'^{be} \gamma_5 + b \gamma_5 A_i \gamma_5 S_d'^{be} S_u^{cf} + \\
& + b \gamma_5 A_i \text{Tr} S_u^{cf} \gamma_5 S_d'^{be} + b A_i S_d'^{be} \gamma_5 S_u^{cf} \gamma_5 + \\
& + b A_i \gamma_5 \text{Tr} S_u^{cf} S_d'^{be} \gamma_5 + \gamma_5 S_u^{cf} \gamma_5 \text{Tr} S_d^{be} A_i' + \\
& + \gamma_5 S_u^{cf} S_d'^{be} A_i \gamma_5 + b^2 S_u^{cf} \gamma_5 S_d'^{be} \gamma_5 A_i + \\
& + b^2 S_u^{cf} \text{Tr} A_i \gamma_5 S_d'^{be} \gamma_5 + b \gamma_5 S_u^{cf} \gamma_5 S_d'^{be} A_i + \\
& + b \gamma_5 S_u^{cf} \text{Tr} A_i \gamma_5 S_d'^{be} + b S_u^{cf} S_d'^{be} \gamma_5 A_i \gamma_5 + \\
& + b S_u^{cf} \gamma_5 \text{Tr} A_i S_d'^{be} \gamma_5 \} + \\
& + \bar{d}^e A_i d^b \{ \gamma_5 S_u^{cf} \gamma_5 \text{Tr} A_i S_u'^{ad} + \\
& + \gamma_5 S_u^{cf} A_i' S_u^{ad} \gamma_5 + b^2 S_u^{cf} \gamma_5 A_i' \gamma_5 S_u^{ad} + \\
& + b^2 S_u^{cf} \text{Tr} S_u^{ad} \gamma_5 A_i' \gamma_5 + b \gamma_5 S_u^{cf} \gamma_5 A_i' S_u^{ad} + \\
& + b \gamma_5 S_u^{cf} \text{Tr} S_u^{ad} \gamma_5 A_i' + b S_u^{cf} A_i' \gamma_5 S_u^{ad} \gamma_5 + \\
& + b S_u^{cf} \gamma_5 \text{Tr} S_u^{ad} A_i' \gamma_5 \} |0\rangle
\end{aligned} \tag{11}$$

where  $A_i = 1$ ,  $\gamma_\alpha$ ,  $\sigma_{\alpha\beta}/\sqrt{2}$ ,  $i\gamma_\alpha\gamma_5$ ,  $\gamma_5$ , a sum over  $A_i$  implied,  $S' \equiv CS^TC$ ,  $A_i' = CA_i^TC$ , with  $T$  denoting the transpose of the matrix, and  $S_q$  is the full quark propagator with both perturbative and non-perturbative contributions. In our calculations, we will neglect the masses of the quarks and assume an  $SU(3)$  flavor symmetry. From Eq. (11) it follows that in order to calculate the correlator (11), the explicit expression of the massless quark propagator is needed. The complete light cone expansion of the light quark

propagator  $S_q$  in external field is given in [14]. It gets contributions from the  $\bar{q}Gq$ ,  $\bar{q}GGq$ ,  $\bar{q}q\bar{q}q$  non-local operators (where  $G$  is the gluon field strength tensor). In this work we consider only operators with one gluon field, corresponding to the quark-antiquark-gluon components of the pion and neglect components with two gluons or four quark fields. This is consistent with the approximation of the twist 4 two particle wave functions obtained in [15]. Taking into account higher Fock-state components would demand corresponding modifications in the two particle wavefunctions via the equations of motion. Formally neglect of the  $\bar{q}GGq$ ,  $\bar{q}q\bar{q}q$  terms can be justified on the basis of an expansion in conformal spin [15]. In this approximation the massless quark propagator is given by:

$$\begin{aligned}
S_q &= \langle 0 | \mathcal{T} \bar{q}(0) q(x) | 0 \rangle \\
&= \frac{i \not{x}}{2\pi^2 x^4} - \frac{\langle \bar{q}q \rangle}{12} - \frac{x^2}{192} m_0^2 \langle \bar{q}q \rangle - \\
&\quad - i g_s \int_0^1 dv \left[ \frac{\not{x}}{16\pi^2 x^2} G_{\mu\nu}(vx) \sigma_{\mu\nu} - vx_\mu G_{\mu\nu}(vx) \gamma_\nu \frac{i}{4\pi^2 x^2} \right] + \dots \quad (12)
\end{aligned}$$

Note that the local part of the propagator consisting of operators with dimension  $d > 5$  is neglected since they give a negligible contribution. In order to evaluate Eq. (11) analytically, one needs the matrix elements of nonlocal operators between the pion state and the vacuum state. The non-zero matrix elements are defined in terms of the pion wave functions up to twist 4 as (see [9, 15, 16]):

$$\begin{aligned}
\langle \pi(q) | \bar{u}(x) \gamma_\mu \gamma_5 u(0) | 0 \rangle = \\
-i f_\pi q_\mu \int_0^1 du e^{iuqx} [\varphi_\pi(u) + x^2 g_1(u)] + f_\pi \left( x_\mu - \frac{x^2 q_\mu}{qx} \right) \int_0^1 du e^{iuqx} g_2(u), \quad (13)
\end{aligned}$$

$$\langle \pi(q) | \bar{u}(x) i \gamma_5 u(0) | 0 \rangle = \frac{f_\pi m_\pi^2}{2m_q} \int_0^1 du e^{iuqx} \varphi_P(u), \quad (14)$$

$$\langle \pi(q) | \bar{u}(x) \sigma_{\mu\nu} \gamma_5 u(0) | 0 \rangle = i(q_\mu x_\nu - q_\nu x_\mu) \frac{f_\pi m_\pi^2}{12m_q} \int_0^1 du e^{iuqx} \varphi_\sigma(u), \quad (15)$$

$$\begin{aligned}
\langle \pi(q) | \bar{u}(x) \sigma_{\alpha\beta} \gamma_5 g_s G_{\mu\nu}(ux) u(0) | 0 \rangle = \\
i f_{3\pi} [(q_\mu q_\alpha g_{\nu\beta} - q_\nu q_\alpha g_{\mu\beta}) - (q_\mu q_\beta g_{\nu\alpha} - q_\nu q_\beta g_{\mu\alpha})] \times \\
\int \mathcal{D}\alpha_i \varphi_{3\pi}(\alpha_i) e^{iqx(\alpha_1 + u\alpha_3)}, \quad (16)
\end{aligned}$$

$$\begin{aligned}
& \langle \pi(q) | \bar{u}(x) \gamma_\mu \gamma_5 g_s G_{\mu\nu}(ux) u(0) | 0 \rangle = \\
& f_\pi \left[ q_\beta \left( g_{\alpha\mu} - \frac{x_\alpha q_\mu}{qx} \right) - q_\alpha \left( g_{\beta\mu} - \frac{x_\beta q_\mu}{qx} \right) \right] \int \mathcal{D}\alpha_i \varphi_\perp(\alpha_i) e^{iqx(\alpha_1+u\alpha_3)} + \\
& + f_\pi \frac{q_\mu}{qx} (q_\alpha x_\beta - q_\beta x_\alpha) \int \mathcal{D}\alpha_i \varphi_\parallel(\alpha_i) e^{iqx(\alpha_1+u\alpha_3)}, \tag{17}
\end{aligned}$$

$$\begin{aligned}
& \langle \pi(q) | \bar{u}(x) \gamma_\mu g_s \tilde{G}_{\mu\nu}(ux) u(0) | 0 \rangle = \\
& i f_\pi \left[ q_\beta \left( g_{\alpha\mu} - \frac{x_\alpha q_\mu}{qx} \right) - q_\alpha \left( g_{\beta\mu} - \frac{x_\beta q_\mu}{qx} \right) \right] \int \mathcal{D}\alpha_i \tilde{\varphi}_\perp(\alpha_i) e^{iqx(\alpha_1+u\alpha_3)} + \\
& + i f_\pi \frac{q_\mu}{qx} (q_\alpha x_\beta - q_\beta x_\alpha) \int \mathcal{D}\alpha_i \tilde{\varphi}_\parallel(\alpha_i) e^{iqx(\alpha_1+u\alpha_3)}. \tag{18}
\end{aligned}$$

Here, the operator  $\tilde{G}_{\alpha\beta}$  is the dual of the gluon field strength tensor,  $\tilde{G}_{\alpha\beta} = \frac{1}{2} \epsilon_{\alpha\beta\delta\rho} G^{\delta\rho}$ ,  $\mathcal{D}\alpha_i$  is defined as  $\mathcal{D}\alpha_i = d\alpha_1 d\alpha_2 d\alpha_3 \delta(1 - \alpha_1 - \alpha_2 - \alpha_3)$ .

Note that the corresponding matrix elements for the  $d$ -quark can be obtained from the  $u$ -quark matrix elements using the isospin relation:

$$\langle \pi | \bar{d}(x) \mathcal{O} d(0) | 0 \rangle = - \langle \pi | \bar{u}(x) \mathcal{O} u(0) | 0 \rangle, \tag{19}$$

where  $\mathcal{O}$  is any of the matrices appearing in Eqs. (13-18).

Note that one can write Eq. (11) as the sum of two terms:

$$\Pi_{\pi NN} = \Pi_1 + \Pi_2 \tag{20}$$

where  $\Pi_1$  is obtained from the terms in the OPE proportional to  $\bar{u} A_i u$  and  $\Pi_2$  is obtained from the terms proportional to  $\bar{d} A_i d$ . With this separation of the terms, and noting that the currents for the other baryons,  $\Sigma$  and  $\Xi$  can be obtained from proton current by simple substitutions, one can immediately obtain the following results for the OPE for the  $\pi\Sigma\Sigma$  and  $\pi\Xi\Xi$  couplings:

$$\Pi_{\pi\Sigma\Sigma} = \Pi_1, \tag{21}$$

$$\Pi_{\pi\Xi\Xi} = -\Pi_2. \tag{22}$$

In deriving Eqs. (21) and (22), we have used the fact that up to twist four the matrix elements of the  $s$ -quark operators between vacuum state and the one pion state are zero.

Extension of the sum rules for the couplings  $\pi NN$  to the  $\eta NN$  coupling is obvious, i.e. it is enough in the correlator (3) to replace  $\pi^0$  by  $\eta$ . Note that in the  $SU(3)$  limit, there is no  $\eta - \eta'$  mixing, and the strange quark component

of  $\eta$  does not participate in the sum rules for  $\eta NN$ , but it gives contribution to the  $\eta\Xi\Xi$  and  $\eta\Sigma\Sigma$  sum rules. In this limit, the matrix elements for  $\eta$  can be obtained from the corresponding matrix elements for  $\pi$  by:

$$\langle\eta|\bar{u}\mathcal{O}u|0\rangle = \langle\eta|\bar{d}\mathcal{O}d|0\rangle = \frac{1}{\sqrt{3}}\langle\pi|\bar{u}\mathcal{O}u|0\rangle \quad (23)$$

$$\langle\eta|\bar{s}\mathcal{O}s|0\rangle = -\frac{2}{\sqrt{3}}\langle\pi|\bar{u}\mathcal{O}u|0\rangle \quad (24)$$

Compared with the  $\pi^0$  case, we see that the sign of the contribution coming from the  $u$  and  $d$  quarks are the same.

The corresponding OPEs can be written as:

$$\Pi_{\eta NN} = \frac{1}{\sqrt{3}}(\Pi_1 - \Pi_2) , \quad (25)$$

$$\Pi_{\eta\Sigma\Sigma} = \frac{1}{\sqrt{3}}(\Pi_1 + 2\Pi_2) , \quad (26)$$

$$\Pi_{\eta\Xi\Xi} = \frac{1}{\sqrt{3}}(-2\Pi_1 - \Pi_2) . \quad (27)$$

Substituting the matrix elements in Eq. (11), one can analytically evaluate the OPE for the correlator. In  $x$  representation for the correlator function we get:

$$\begin{aligned} \Pi_1(p_1^2, p_2^2) &= \int d^4x du e^{ipx} \left\{ e^{-iuqx} \times \right. \\ &- \left\{ \frac{f_\pi}{\pi^4 x^6} \left[ (3 + 2b + 3b^2)\gamma_5 \not{x} + 4b \frac{qx}{x^2} \gamma_5 \not{x} \right] (\varphi_\pi(u) + x^2 g_1(u)) \right. \\ &+ \frac{f_\pi}{\pi^4 x^6} g_2(u) (3 + 2b + 3b^2) i \left( \gamma_5 \not{x} - \frac{x^2}{qx} \gamma_5 \not{x} \right) - \\ &- \frac{f_\pi}{288\pi^2 x^4} \mu \langle \bar{q}q \rangle \left( 16(b - b^2) + \frac{1}{6}(-1 + 3b - 2b^2)m_0^2 x^2 \right) \times \\ &\times \varphi_\sigma(u) \left[ x^2 \gamma_5 \not{x} - qx \gamma_5 \not{x} \right] \Big\} + \\ &+ \int \mathcal{D}\alpha_i e^{-i(\alpha_1 + \bar{u}\alpha_3)qx} \left\{ \right. \\ &i \frac{f_{3\pi}}{24\pi^2 x^2} \langle \bar{q}q \rangle (b - b^2)(1 - 2u) \varphi_{3\pi}(\alpha_i) \left[ 2qx \gamma_5 \not{x} + q^2 \gamma_5 \not{x} \right] + \\ &+ \frac{f_\pi}{2\pi^4 x^6} b(1 - 2u) \varphi_\parallel(\alpha_i) \left[ -x^2 \gamma_5 \not{x} + 2qx \gamma_5 \not{x} - q^2 \frac{x^2}{qx} \gamma_5 \not{x} \right] - \end{aligned}$$



$$\begin{aligned}
& - \frac{f_\pi}{2\pi^4 x^6} [\varphi_\perp(\alpha_i)(1-2u) - \tilde{\varphi}_\perp(\alpha_i)] b \left( 2qx + q^2 \frac{x^2}{qx} \right) \gamma_5 \not{x} + \\
& + \frac{f_\pi}{4\pi^4 x^4} (b^2 + 1) \left( \gamma_5 \not{x} - \frac{qx}{x^2} \gamma_5 \not{x} \right) \tilde{\varphi}_\parallel(\alpha_i) + \\
& + \frac{f_\pi}{2\pi^4 x^4} \frac{q^2}{qx} \tilde{\varphi}_\parallel(\alpha_i) b \gamma_5 \not{x} \Big\} \Big\} + \\
& + \sigma_{\alpha\beta} \gamma_5 q^\alpha \int d^4 x du e^{ipx} x^\beta \left\{ e^{-iuqx} \right. \\
& \quad \left\{ \frac{f_\pi}{48\pi^2 x^4} \langle \bar{q}q \rangle \left( 16(b-b^2) + \frac{1}{6}(-1+3b-2b^2)m_0^2 x^2 \right) \times \right. \\
& \quad \times [\varphi_\pi(u) + x^2 g_1(u)] + \\
& \quad + i \frac{f_\pi}{3\pi^2 x^2} (b-b^2) \langle \bar{q}q \rangle \frac{g_2(u)}{qx} - \frac{f_\pi}{2\pi^4 x^6} \mu(b^2-1) \varphi_\sigma(u) \Big\} + \\
& \quad + \int \mathcal{D}\alpha_i e^{-i(\alpha_1 + \bar{u}\alpha_3)qx} \left\{ -\frac{f_\pi}{24\pi^2 x^2} (b-b^2) \langle \bar{q}q \rangle (1-2u) \varphi_\parallel(\alpha_i) - \right. \\
& \quad \left. - \frac{f_\pi}{24\pi^2 x^2} \langle \bar{q}q \rangle (b-1) \tilde{\varphi}_\parallel(\alpha_i) \Big\} \Big\} \tag{28}
\end{aligned}$$

$$\begin{aligned}
\Pi_2(p_1^2, p_2^2) = & \int d^4 x du e^{ipx} \left\{ e^{-iuqx} \times \right. \\
& \left\{ -\frac{f_\pi}{2\pi^4 x^6} \left[ (b+1)^2 \gamma_5 \not{x} - 2(1+6b+b^2) \frac{qx}{x^2} \gamma_5 \not{x} \right] (\varphi_\pi(u) + x^2 g_1(u)) + \right. \\
& + \frac{f_\pi}{2\pi^4 x^6} g_2(u) (b+1)^2 i \left( \gamma_5 \not{x} - \frac{x^2}{qx} \gamma_5 \not{x} \right) + \\
& + \frac{f_\pi}{576\pi^2 x^4} \mu \langle \bar{q}q \rangle \left( 16 + \frac{7}{6} m_0^2 x^2 \right) (b^2-1) \varphi_\sigma(u) \left[ x^2 \gamma_5 \not{x} - qx \gamma_5 \not{x} \right] \Big\} + \\
& + \int \mathcal{D}\alpha_i e^{-i(\alpha_1 + \bar{u}\alpha_3)qx} \left\{ \right. \\
& \quad -i \frac{f_{3\pi}}{48\pi^2 x^2} \langle \bar{q}q \rangle (b^2-1)(1-2u) \varphi_{3\pi}(\alpha_i) \left[ 2qx \gamma_5 \not{x} + q^2 \gamma_5 \not{x} \right] - \\
& \quad - \frac{f_\pi}{8\pi^4 x^6} (1+6b+b^2)(1-2u) \varphi_\parallel(\alpha_i) \left[ -x^2 \gamma_5 \not{x} + 2qx \gamma_5 \not{x} - q^2 \frac{x^2}{qx} \gamma_5 \not{x} \right] + \\
& \quad + \frac{f_\pi}{8\pi^4 x^6} (1+6b+b^2) [\varphi_\perp(\alpha_i)(1-2u) - \tilde{\varphi}_\perp(\alpha_i)] \left( 2qx + q^2 \frac{x^2}{qx} \right) \gamma_5 \not{x} -
\end{aligned}$$

$$\begin{aligned}
& - \frac{f_\pi}{8\pi^4 x^4} \left[ (3 + 2b + 3b^2) \gamma_5 \not{x} - 4(b+1)^2 \frac{qx}{x^2} \gamma_5 \not{x} \right] \tilde{\varphi}_\parallel(\alpha_i) - \\
& - \frac{f_\pi}{8\pi^2 x^4} \frac{q^2}{qx} \tilde{\varphi}_\parallel(\alpha_i) (1 + 6b + b^2) \gamma_5 \not{x} \Big\} \Big\} + \\
& + \sigma_{\alpha\beta} \gamma_5 q^\alpha \int d^4 x du e^{ipx} x^\beta \left\{ e^{-iuqx} \right. \\
& \quad \left\{ - \frac{f_\pi}{96\pi^2 x^4} \langle \bar{q}q \rangle (b^2 - 1) \left( 16 + \frac{7}{6} m_0^2 x^2 \right) [\varphi_\pi(u) + x^2 g_1(u)] - \right. \\
& \quad - i \frac{f_\pi}{6\pi^2 x^2} (b^2 - 1) \langle \bar{q}q \rangle \frac{g_2(u)}{qx} - \frac{f_\pi}{12\pi^4 x^6} \mu (b-1)^2 \varphi_\sigma(u) \Big\} + \\
& \quad + \int \mathcal{D}\alpha_i e^{-i(\alpha_1 + \bar{u}\alpha_3)qx} \left\{ \frac{f_\pi}{48\pi^2 x^2} (b^2 - 1) \langle \bar{q}q \rangle (1 - 2u) \varphi_\parallel(\alpha_i) - \right. \\
& \quad - \frac{f_\pi}{48\pi^2 x^2} \langle \bar{q}q \rangle (b^2 - 1) \tilde{\varphi}_\parallel(\alpha_i) + \\
& \quad \left. \left. + i \frac{f_{3\pi}}{4\pi^4 x^6} (b-1)^2 (1 - 2u) qx \varphi_{3\pi}(\alpha_i) \right\} \right\} \quad (29)
\end{aligned}$$

where in Eqs. (28)-(29) we have neglected terms containing the gluon condensate  $\langle g^2 G^2 \rangle$  as it gives a negligible contribution to the sum rules.

Our next problem is to perform Fourier transformation from  $x$  to momentum representations and then, to get the result for the theoretical part of the sum rules, apply on the obtained result double Borel transformations on the variables  $p_1^2$  and  $p_2^2$  in order to suppress the contributions of higher states and the continuum. (the details can be found in [16-19]). In calculating the Fourier transforms of terms containing factors  $qx$  or  $1/qx$ , we have performed integration by parts:

$$\int_0^1 du qx f(u) e^{-iuqx} = -i \int_0^1 du f'(u) e^{-iuqx} + f(u) e^{-iuqx} \Big|_0^1 \quad (30)$$

$$\int_0^1 du \frac{e^{-iuqx}}{qx} g_2(u) = -i \int_0^1 du e^{-iuqx} G_2(u) - \frac{G_2(u)}{qx} e^{-iuqx} \Big|_0^1, \quad (31)$$

where  $f'(u) = \frac{df}{dx}(u)$  and

$$G_2(u) = - \int_0^u du' g_2(u'). \quad (32)$$

The second term in Eq. (31) vanishes since  $G_2(0) = G_2(1) = 0$ .

After performing double Borel transformation over the variable  $p_1^2 = p^2$  and  $p_2^2 = (p + q)^2$  for the  $i\gamma_5 \not{q}$  structure, we obtain the following results for the theoretical part of the sum rules:

$$\begin{aligned}
\Pi_1^\gamma(M^2) = & -\frac{f_\pi}{8\pi^2} M^6 f_2(x) [(3 + 2b + 3b^2) \phi_\pi(u_0) - \frac{2b}{3} u_0 \phi'_\pi(u_0)] + \\
& + (b^2 + 1) u_0 \phi'_\pi(u_0) + \\
& + \frac{f_\pi}{\pi^2} M^4 f_1(x) (3 + 2b + 3b^2) [g_1(u_0) + G_2(u_0)] - \\
& - \frac{f_\pi}{4\pi^2} u_0 g_2(u_0) M^4 f_1(x) (3 + 2b + 3b^2) g_2(u_0) - \\
& - \frac{2f_\pi \mu}{9} \langle \bar{q}q \rangle (b - b^2) M^2 f_0(u_0) \left[ \phi_\sigma(u_0) + \frac{u_0}{2} \phi'_\sigma(u_0) \right] + \\
& + \frac{f_\pi \mu}{216} m_0^2 \langle \bar{q}q \rangle (-1 + 3b - b^2) u_0 \phi'_\sigma(u_0) - \\
& - \frac{f_\pi}{\pi^2} M^4 f_1(x) u_0 g'_1(u_0) b - \\
& - \frac{f_\pi}{4\pi^2} M^4 f_1(x) \left[ u_0 I_1(1 - 2u, \phi_\parallel) + 2I_{11}(1 - 2u, \phi_\parallel) \right] b - \\
& - \frac{f_{3\pi}}{3} \langle \bar{q}q \rangle M^2 f_0(x) I_1(1 - 2u, \phi_{3\pi}) (b - b^2) + \\
& + \frac{f_\pi}{4\pi^2} M^4 f_1(x) u_0 \left[ I_1(1 - 2u, \phi_\perp) + I_1(1, \tilde{\phi}_\perp) \right] b - \\
& - \frac{f_\pi}{16\pi^2} M^4 f_1(x) \left[ (b + 1)^2 u_0 I_1(1, \tilde{\phi}_\parallel) + 4(b^2 + 1) I_{11}(1, \tilde{\phi}_\parallel) \right] + \\
& + \frac{f_\pi}{4\pi^2} M^4 f_1(x) u_0 I_1(1, \tilde{\phi}_\perp) b \tag{33} \\
\Pi_2^\gamma(M^2) = & -\frac{f_\pi}{48\pi^2} M^6 f_2(x) \left[ 3(1 + b)^2 \phi_\pi(u_0) + (1 + 6b + b^2) u_0 \phi'_\pi(u_0) \right] + \\
& + \frac{f_\pi}{2\pi^2} M^4 f_1(x) [g_1(u_0) + G_2(u_0)] (b + 1)^2 - \\
& - \frac{f_\pi}{8\pi^2} M^4 f_1(x) u_0 g_2(u_0) (b + 1)^2 + \\
& + \frac{f_\pi}{9} \mu \langle \bar{q}q \rangle M^2 f_0(x) (b^2 - 1) \left[ \phi_\sigma(u_0) + \frac{u_0}{2} \phi'_\sigma(u_0) \right] - \\
& - \frac{7f_\pi \mu}{432} m_0^2 \langle \bar{q}q \rangle u_0 \phi'_\sigma(u_0) (b^2 - 1) + \\
& + \frac{f_\pi}{4\pi^2} M^4 f_1(x) u_0 g'_1(u_0) (1 + 6b + b^2) +
\end{aligned}$$

$$\begin{aligned}
& + \frac{f_\pi}{16\pi^2} M^4 f_1(x) (1 + 6b + b^2) \left[ u_0 I_1(1 - 2u, \phi_\parallel) + 2I_{11}(1 - 2u, \phi_\parallel) \right] + \\
& + \frac{f_{3\pi}}{6} \langle \bar{q}q \rangle M^2 f_0(x) (b^2 - 1) I_1(1 - 2u, \phi_{3\pi}) - \\
& - \frac{f_\pi}{16\pi^2} M^4 f_1(x) (1 + 6b + b^2) I_1(1 - 2u, \phi_\perp) + \\
& + \frac{f_\pi}{8\pi^2} M^4 f_1(x) \left[ (1 + b)^2 u_0 I_1(1, \tilde{\phi}_\parallel) + (3 + 2b + 3b^2) I_{11}(1, \tilde{\phi}_\parallel) \right] - \\
& - \frac{f_\pi}{16\pi^2} M^4 f_1(x) (1 + 6b + b^2) u_0 I_1(1, \tilde{\phi}_\perp). \tag{34}
\end{aligned}$$

Similarly for the  $\sigma_{\mu\nu} \gamma_5 p^\mu q^\nu$  structure we get:

$$\begin{aligned}
\Pi_1^\sigma(M^2) &= \frac{2f_\pi}{3\pi^2} \langle \bar{q}q \rangle \left[ M^2 f_0(x) (b - b^2) - \frac{m_0^2}{24} (-1 + 3b - b^2) \right] \phi_\pi(u_0) - \\
&- \frac{8f_\pi}{3} \langle \bar{q}q \rangle (b - b^2) [g_1(u_0) + G_2(u_0)] + \\
&+ \frac{f_\pi}{8\pi^2} \mu M^4 f_1(x) (b^2 - 1) \phi_\sigma(u_0) - \\
&- \frac{f_\pi}{3} \langle \bar{q}q \rangle (b - b^2) I_{11}(1 - 2u, \phi_\parallel) + \\
&+ \frac{f_\pi}{3} \langle \bar{q}q \rangle (b - 1) I_{11}(1, \tilde{\phi}_\parallel) \tag{35}
\end{aligned}$$

$$\begin{aligned}
\Pi_2^\sigma(M^2) &= -\frac{f_\pi}{3} \langle \bar{q}q \rangle \left[ M^2 f_0(x) - \frac{7m_0^2}{24} \right] (b^2 - 1) \phi_\pi(u_0) + \\
&+ \frac{4f_\pi}{3} \langle \bar{q}q \rangle (b^2 - 1) [g_1(u_0) + G_2(u_0)] + \\
&+ \frac{f_\pi}{48\pi^2} \mu M^4 f_1(x) (b - 1)^2 \phi_\sigma(u_0) + \\
&+ \frac{f_\pi}{6} \langle \bar{q}q \rangle (b^2 - 1) I_{11}(1 - 2u, \phi_\parallel) - \\
&+ \frac{f_{3\pi}}{16\pi^2} M^4 f_1(x) (b - 1)^2 I_{11}(1 - 2u, \phi_{3\pi}) + \\
&+ \frac{f_\pi}{6} \langle \bar{q}q \rangle (b^2 - 1) I_{11}(1, \tilde{\phi}_\parallel) \tag{36}
\end{aligned}$$

where the superscripts  $\gamma$  and  $\sigma$  corresponds to the  $i\gamma_5 \not{q}$  and  $\sigma_{\mu\nu} \gamma_5 p^\mu q^\nu$  structures respectively,  $\mu = \frac{m_\pi^2}{2m_q} = -\frac{\langle \bar{q}q \rangle}{f_\pi^2}$ ,  $x = \frac{s_0}{M^2}$ ,  $f_n(x) = 1 - e^{-x}(1 + x +$

$\frac{x^2}{2} + \dots + \frac{x^n}{n!}$ ,  $s_0$  is the continuum threshold,  $u_0 = M_2^2/(M_1^2 + M_2^2)$ ,

$$\frac{1}{M^2} = \frac{1}{M_1^2} + \frac{1}{M_2^2} \quad (37)$$

and the function  $I_1$  and  $I_{11}$  are defined as:

$$I_{11}(f(u), g(\alpha_i)) = \int_0^{u_0} d\alpha_1 \int_0^{1-u_0} d\alpha_2 \frac{f(\frac{u_0-\alpha_1}{1-\alpha_1-\alpha_2})}{1-\alpha_1-\alpha_2} g(\alpha_1, \alpha_2, 1-\alpha_1-\alpha_2) \quad (38)$$

$$\begin{aligned} I_1(f(u), g(\alpha_i)) &= \int_0^{u_0} d\alpha_1 \int_0^{1-u_0} d\alpha_2 \frac{f(\frac{u_0-\alpha_1}{1-\alpha_1-\alpha_2})}{1-\alpha_1-\alpha_2} g(\alpha_1, \alpha_2, 1-\alpha_1-\alpha_2) + \\ &+ \int_0^{1-u_0} d\alpha_2 \frac{f(0)}{1-u_0-\alpha_2} g(u_0, \alpha_2, 1-u_0-\alpha_2) - \\ &- \int_0^{u_0} d\alpha_1 \frac{f(1)}{u_0-\alpha_1} g(\alpha_1, 1-u_0, u_0-\alpha_1) \end{aligned} \quad (39)$$

Equating theoretical (see Eqs. (33-36)) and phenomenological (see Eq. (7)) parts of the correlator function (3) we arrive at the following sum rules for the meson baryon coupling constants:

a) For the  $i\gamma_5 \not{q}$  structure:

$$\begin{aligned} -m_N \lambda_N^2 g_{\pi NN} e^{-\frac{m_N^2}{M^2}} &= \Pi_1^\gamma(M^2) + \Pi_2^\gamma(M^2) \\ -m_\Sigma \lambda_\Sigma^2 g_{\pi \Sigma \Sigma} e^{-\frac{m_\Sigma^2}{M^2}} &= \Pi_1^\gamma(M^2) \\ -m_\Xi \lambda_\Xi^2 g_{\pi \Xi \Xi} e^{-\frac{m_\Xi^2}{M^2}} &= -\Pi_2^\gamma(M^2) \\ -\sqrt{3} m_N \lambda_N^2 g_{\eta NN} e^{-\frac{m_N^2}{M^2}} &= \Pi_1^\gamma(M^2) - \Pi_2^\gamma(M^2) \\ -\sqrt{3} m_\Sigma \lambda_\Sigma^2 g_{\eta \Sigma \Sigma} e^{-\frac{m_\Sigma^2}{M^2}} &= \Pi_1^\gamma(M^2) + 2\Pi_2^\gamma(M^2) \\ -\sqrt{3} m_\Xi \lambda_\Xi^2 g_{\eta \Xi \Xi} e^{-\frac{m_\Xi^2}{M^2}} &= -2\Pi_1^\gamma(M^2) - \Pi_2^\gamma(M^2) \end{aligned} \quad (40)$$

b) For the  $\sigma_{\mu\nu} \gamma_5 p^\mu q^\nu$  structure:

$$\lambda_N^2 g_{\pi N} e^{-\frac{m_N^2}{M^2}} = \Pi_1^\sigma(M^2) + \Pi_2^\sigma(M^2)$$

$$\begin{aligned}
\lambda_{\Sigma}^2 g_{\pi\Sigma\Sigma} e^{-\frac{m_{\Sigma}^2}{M^2}} &= \Pi_1^{\sigma}(M^2) \\
\lambda_{\Xi}^2 g_{\pi\Xi\Xi} e^{-\frac{m_{\Xi}^2}{M^2}} &= -\Pi_2^{\sigma}(M^2) \\
\sqrt{3}\lambda_N^2 g_{\eta N} e^{-\frac{m_N^2}{M^2}} &= \Pi_1^{\sigma}(M^2) - \Pi_2^{\sigma}(M^2) \\
\sqrt{3}\lambda_{\Sigma}^2 g_{\eta\Sigma\Sigma} e^{-\frac{m_{\Sigma}^2}{M^2}} &= \Pi_1^{\sigma}(M^2) + 2\Pi_2^{\sigma}(M^2) \\
\sqrt{3}\lambda_{\Xi}^2 g_{\eta\Xi\Xi} e^{-\frac{m_{\Xi}^2}{M^2}} &= -2\Pi_1^{\sigma}(M^2) - \Pi_2^{\sigma}(M^2)
\end{aligned} \tag{41}$$

These results are obtained in the  $SU(3)$  limit, i.e. all masses and residues of the baryons are the same and  $\beta = \langle \bar{q}q \rangle$  ( $q = u, d$ ). Note also that Eqs. (40) and (41) are consistent with the  $SU(3)$  symmetry relations Eq. (2). In terms of the OPE, the F/D ratio can be identified as:

$$2\alpha = \frac{\Pi_1(M^2)}{\Pi_1(M^2) + \Pi_2(M^2)} \rightarrow F/D = \frac{\Pi_1(M^2)}{\Pi_1(M^2) + 2\Pi_2(M^2)} \tag{42}$$

### 3 Numerical Analysis

In this section we analyze the sum rules obtained in the previous section for the coupling constants in the  $SU(3)$  limit and study the dependence of the  $F/D$  ratio on the Dirac structure. Since the coupling constants are physical quantities, they should be independent of the parameter  $b$  and the continuum threshold  $s_0$ . Therefore our first problem is to find the region in the parameter space where they are practically independent of  $b$  and  $s_0$ . In  $SU(3)$  limit, all baryon masses and their residues are equal and also  $\langle q\bar{q} \rangle = \beta$  and  $f_{\eta} = f_{\pi}$ .

The main input parameters of the sum rules (see Eqs. (33-36) and (40-41)), are the pion wave functions. In [20], a theoretical framework has been developed to study these functions. The leading twist 2 pion wave function can be expressed as an expansion in Gegenbauer polynomials  $C_i^{3/2}$  [9]:

$$\varphi_{\pi}(u) = 6u(1-u) \left[ 1 + a_2 C_2^{3/2}(2u-1) + a_4 C_4^{3/2} + \dots \right] \tag{43}$$

The coefficients  $a_i$  renormalize multiplicatively. On the basis of the approximate conformal symmetry of QCD, it has been shown in [15] that the expansion (43) converges sufficiently fast so that terms with  $n > 4$  are negligible.

In the calculations we have used the following forms of the wavefunctions appearing in the meson matrix elements (see e.g. [9, 15] for more details):

$$\begin{aligned}
\varphi_\sigma(u, \mu) &= 6u\bar{u} \left[ 1 + C_2 \frac{3}{2} (5(2u-1)^2 - 1) \right. \\
&\quad \left. + C_4 \frac{15}{8} (21(2u-1)^4 - 14(2u-1)^2 + 1) \right] , \\
\varphi_\pi(u) &= 6u(1-u) \left[ 1 + a_2 \frac{3}{2} (5(2u-1)^2 - 1) + a_4 \frac{15}{8} (21(2u-1)^4 - 14(2u-1)^2 + 1) \right] \\
\varphi_P(u) &= 1 + B_2 \frac{1}{2} (3(2u-1)^2 - 1) + B_4 \frac{1}{8} (35(2u-1)^4 \\
&\quad - 30(2u-1)^2 + 3) , \\
g_1(u) &= \frac{5}{2} \delta^2 (1-u)^2 u^2 + \frac{1}{2} \varepsilon \delta^2 [(1-u)u(2+13(1-u)u) + 10u^3 \ln u(2-3u + \frac{6}{5}u^2) \\
&\quad + 10(1-u)^3 \ln(1-u)(2-3(1-u) + \frac{6}{5}(1-u)^2)] , \\
g_2(u) &= \frac{10}{3} \delta^2 u(1-u)(2u-1) \\
\varphi_\parallel(\alpha_i) &= 120\delta^2 \varepsilon (\alpha_1 - \alpha_2) \alpha_1 \alpha_2 \alpha_3 \\
\varphi_\perp(\alpha_i) &= 10\delta^2 (\alpha_1 - \alpha_2) \alpha_3^2 [1 + 6\varepsilon(1-2\alpha_3)] \\
\tilde{\varphi}_\parallel(\alpha_i) &= -40\delta^2 \alpha_1 \alpha_2 \alpha_3 [1 + 3\varepsilon(1-3\alpha_3)] \\
\tilde{\varphi}_\perp(\alpha_i) &= 10\delta^2 (1-\alpha_3) \alpha_3^2 [1 + 6\varepsilon(1-2\alpha_3)] \\
\varphi_{3\pi}(\alpha_i) &= 360\alpha_1 \alpha_2 \alpha_3^2
\end{aligned} \tag{44}$$

where  $\delta$  is defined by matrix element:

$$\langle \pi | g_s \bar{q} \tilde{G}_{\alpha\mu} \gamma^\alpha q | 0 \rangle = i\delta^2 f_\pi q_\mu . \tag{45}$$

and

$$\begin{aligned}
B_2 &= 30 \frac{f_{3\pi}}{\mu f_\pi} , \quad B_4 = \frac{3}{2} \frac{f_{3\pi}}{\mu f_\pi} (4\omega_{2,0} - \omega_{1,1} - 2\omega_{1,0}) , \\
C_2 &= \frac{f_{3\pi}}{\mu f_\pi} (5 - \frac{1}{2} \omega_{1,0}) , \quad C_4 = \frac{1}{10} \frac{f_{3\pi}}{\mu f_\pi} (4\omega_{2,0} - \omega_{1,1}) .
\end{aligned} \tag{46}$$

The additional parameters appearing in the above are numerically given by

$$\omega_{1,0} = -2.88 , \quad \omega_{2,0} = 10.5 , \quad \omega_{1,1} = 0 , \quad \varepsilon = 0.5 , \quad a_2 = \frac{2}{3} , \quad a_4 = 0.43 ,$$

which corresponds to choosing the renormalization scale at  $1 \text{ GeV}$ .

In our calculations, we set  $M_1^2 = M_2^2 = 2M^2$  since the initial and final baryons are identical, which corresponds to setting  $u_0 = 1/2$ . Hence, in the sum rules, only the value of the wavefunctions at the symmetry point  $u = 1/2$  are needed.

The values of the other input parameters appearing in the sum rules are:  $f_\pi = 0.013 \text{ GeV}$ ,  $\langle g^2 G^2 \rangle = 0.474 \text{ GeV}^4$ ,  $\langle \bar{q}q \rangle = -(0.243)^3 \text{ GeV}^3$ ,  $m_0^2 = (0.8 \pm 0.2) \text{ GeV}^2$  [21],  $\delta^2 = 0.2 \text{ GeV}^2$  [22],  $f_{3\pi} = 0.0035 \text{ GeV}^2$ ,  $\mu = 1.8797$ .

In Figs. (1) and (2), we present the dependence of  $g_{\pi NN} \lambda_N^2(b)$  on the Borel mass  $M^2$  at three different values of  $b$ , namely  $b = -1.5$ ,  $b = 1.5$  as well as  $b = -1$  which corresponds to the Ioffe current for the structures  $i\gamma_5 \not{q}$  and  $\sigma_{\mu\nu} \gamma_5 p^\mu q^\nu$  respectively. The continuum threshold is chosen to be  $s_0 = 2.07 \text{ GeV}^2$  corresponding to the Roper resonance. To analyze the sensitivity of the sum rules to the continuum threshold, we also plotted the results for the value  $s_0 = 2.57 \text{ GeV}^2$ . From these figures we obtained that the working region for the Borel mass is  $0.6 \leq M^2 \leq 1.2, \text{ GeV}^2$ . Also we see that the tensor structure  $\sigma_{\mu\nu} \gamma_5 p^\mu q^\nu$  is more stable with respect to variations of the Borel mass and also the variations of the continuum threshold for all curves. For example at  $M^2 = 1 \text{ GeV}^2$ , the results are practically independent of  $s_0$  and  $b$ . The results change about  $\sim 5\%$  for the structure  $\sigma_{\mu\nu} \gamma_5 p^\mu q^\nu$  with variations of  $s_0$  and  $b$ . This indicates that the results obtained from the structure  $i\gamma_5 \not{q}$  are less reliable.

In Figs (3) and (4), the dependence of  $g_{\mathcal{M}BB} \lambda_B^2(b)$  on the parameter  $b$  at  $M^2 = 1 \text{ GeV}^2$  and at  $s_0 = 2.07 \text{ GeV}^2$  is presented for the structures  $i\gamma_5 \not{q}$  and  $\sigma_{\mu\nu} \gamma_5 p^\mu q^\nu$ . The coupling constants  $g_{\mathcal{M}BB}$  are physical parameters and hence it should not depend on the arbitrary parameter  $b$ . Since  $\lambda_B$ 's are the same for all baryons due to the  $SU(3)$  symmetry, one expects the graphs to be just multiples of one another. The results obtained from the structure  $i\gamma_5 \not{q}$  are far from satisfying this criteria. On the other hand the results obtained from the  $\sigma_{\mu\nu} \gamma_5 p^\mu q^\nu$  structure are all zero at  $b = 1$  similar to the traditional sum rules (see [11]) but the second zero position is different for different coupling constants. Note that the region  $-0.5 \leq b \leq 1$  is unphysical since in this region the mass sum rules yields a negative value for  $\lambda_B^2(b)$  [11]. As we see from Fig. 4, for each sum rule, this unphysical region becomes wider and contains the region between the two zeroes, since the sign of  $g_{\mathcal{M}BB} \lambda_B^2(b)$  should be the same as the sign of  $g_{\mathcal{M}BB}$ , hence it should not change. From these figures also one is led to the conclusion that the predictions of the  $i\gamma_5 \not{q}$  structure is not as reliable as the predictions of the  $\sigma_{\mu\nu} \gamma_5 p^\mu q^\nu$  structure in



constructing sum rules.

In Fig (5), the dependence of the  $F/D$  ratio for the above mentioned structures on  $\cos\theta$  is presented. Here,  $\theta$  is defined as  $b = \tan\theta$  and only the physical region for the parameter  $\theta$  is shown. From these figures, it follows that for the tensor structure, the ratio  $F/D$  is practically independent on the continuum threshold, but for the  $i\gamma_5 \not{q}$  structure, it has a strong dependence on  $s_0$ . For this reason, prediction for the  $F/D$  ratio from the tensor structure is more reliable. The dependence of the  $F/D$  ratio on the Borel parameter  $M^2$  also turns out to be very weak. Being a physical parameter, the  $F/D$  ratio should be independent of  $\cos\theta$ . From the figure, we see that  $F/D$  is quite stable (for the tensor structure) in the region  $-0.25 \leq \cos\theta \leq 0.50$ . This region is also away from the unphysical region for  $b$ . In this region, we obtain the result  $F/D = 0.6 \pm 0.1$ . Note that  $SU(6)$  quark model predict  $F/D = 2/3$ . Analysis of semileptonic decay of hyperons give  $F/D \simeq 0.57$  [23] and the traditional sum rules yield  $0.6 \leq F/D \leq 0.8$  [11]. Within errors our results are in a good agreement with all existing results.

## References

- [1] M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Nucl Phys. **B147**, 385 (1979)
- [2] H. Shoimi and T. Hatsuda, Nucl. Phys. **A594** (1995) 294
- [3] M. C. Birse and B. Krippa, Phys. Lett. **B73** (1996) 9
- [4] H. Kim, S. H. Lee and M. Oka, Phys. Lett **B453** (1999) 199; Phys. Rev. **D60** (1999) 034007
- [5] H. Kim, Eur. Phys. Jour. **A7** (2000) 121
- [6] H. Kim, T. Doi, M. Oka and S. H. Lee, Nucl. Phys. **A662** (2000) 371
- [7] H. Kim, T. Doi, M. Oka and S. H. Lee, Nucl. Phys. **A678** (2000) 295
- [8] L. J. Reinders, H. Rubinstein and S. Yazaki, Phys. Rep. **127C** (1985) 1
- [9] V.M. Braun and I.E. Filyanov, Z.Phys. **C44** (1989) 157, Sov.J.Nucl.Phys. **50** (1989) 511
- [10] J. J. de Swart, Rev. Mod. Phys **35** (1963) 916; **37** (1965) 326 (E)
- [11] T. Doi, H. Kim, M. Oka, Phys. Rev. **C62** (2000) 055202
- [12] V. M. Braun, preprint, hep-ph/9801222; Proc. Rostock 1997, "Progress on Heavy Quark Physics," p.105 (1997)
- [13] V. M. Braun, preprint, hep-ph/9911206 (1999)
- [14] I. I. Balitsky and V.M. Braun, Nucl.Phys. **B311** (1988) 541
- [15] V.M. Braun and I.E. Filyanov, Z.Phys. **C48** (1990) 239
- [16] V. M. Belyaev, V. M. Braun, A. Khodjamirian and R. Rückl, Phys. Rev. **D51** (1995) 6177
- [17] T. M. Aliev, M. Savci, Phys. Rev. **D61** (2000) 016008
- [18] T. M. Aliev, A. Ozpineci, M. Savci, Phys.Rev. **D62** (2000) 053012
- [19] T. M. Aliev, A. Ozpineci, M. Savci, Nucl.Phys. **A678** (2000) 443

- [20] V. L. Chernyak and A. R. Zhitnitsky, JETP Lett. **25** (1997) 510; Sov. J. Nucl. Phys. **31** (1980) 544; A. V. Efremov and A. V. Radyushkin, Phys. Lett. **B94** (1980) 245; Teor. Mat. Fiz. **42** (1980) 147; G. P. Lepage and S. J. Brodsky, Phys. Lett. **B87** (1979) 359; Phys. Rev. **D22** (1980) 2157
- [21] V. M. Belyaev and B. L. Ioffe, JETP **56** 493,1982
- [22] V. A. Novikov, M. A. Shifman, A.I. Vainshtein, M. B. Voloshin and V. I. Zakharov, Nucl. Phys. **B237** (1984) 525
- [23] P.G. Ratcliffe, Phys. Lett. **B365** (1996) 383

## Figure Captions

Fig. 1. The dependence of  $g_{\pi NN}\lambda_N^2(b)$  on the Borel mass,  $M^2$  at  $b = -1.5, -1.0, 1.5$  and at the continuum threshold  $s_0 = 2.07 \text{ GeV}^2$  and  $s_0 = 2.57 \text{ GeV}^2$  for the  $i\gamma_5 \not{q}$  structure.

Fig. 2. The same as Fig. 1 but for the structure  $\sigma_{\mu\nu}\gamma_5 p^\mu q^\nu$

Fig. 3. The dependence of  $g_{\mathcal{M}BB}\lambda_B^2(b)$  on  $b$  at  $M^2 = 1 \text{ GeV}^2$  and the continuum threshold  $s_0 = 2.07 \text{ GeV}^2$  for the  $i\gamma_5 \not{q}$  structure.

Fig. 4. The same as Fig. (3) but for the structure  $\sigma_{\mu\nu}\gamma_5 p^\mu q^\nu$

Fig. 5. The dependence of F/D on  $\theta$  at  $M^2 = 1 \text{ GeV}^2$  for both structures at two values of the continuum threshold,  $s_0 = 2.07 \text{ GeV}^2$  and  $s_0 = 2.57 \text{ GeV}^2$ .

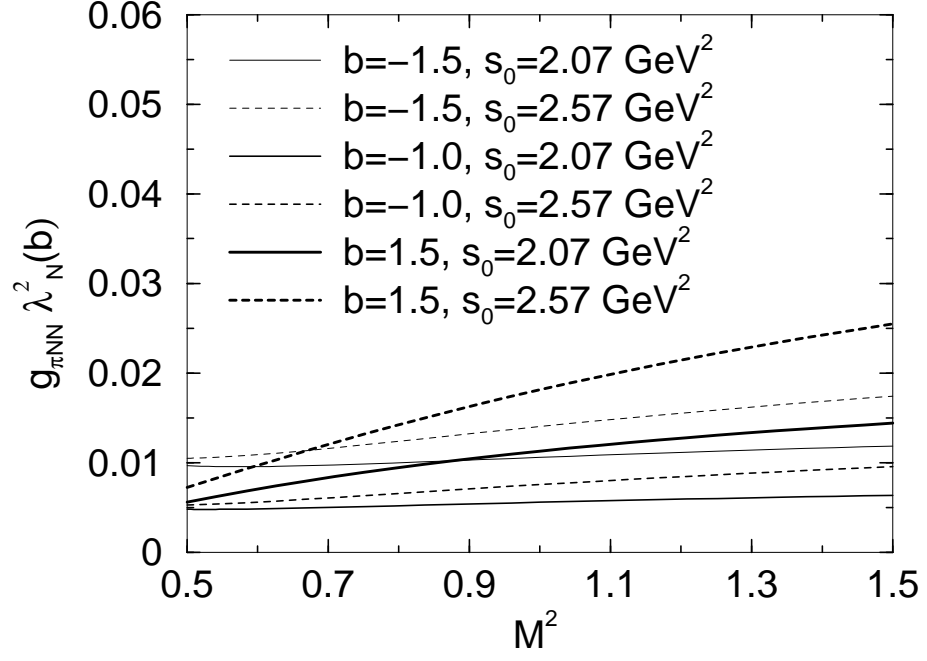


Figure 1:

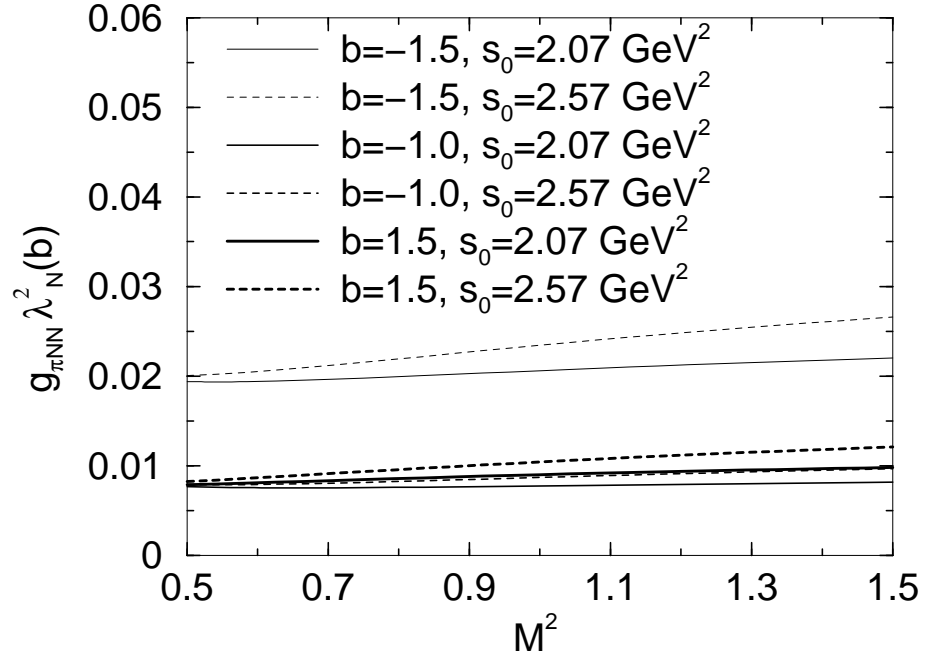


Figure 2:

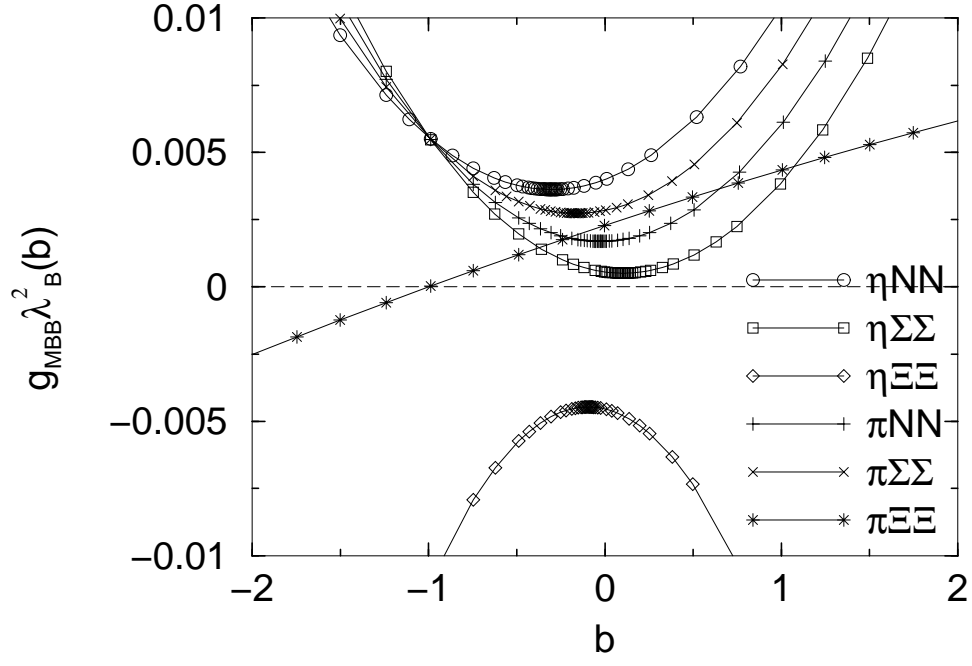


Figure 3:

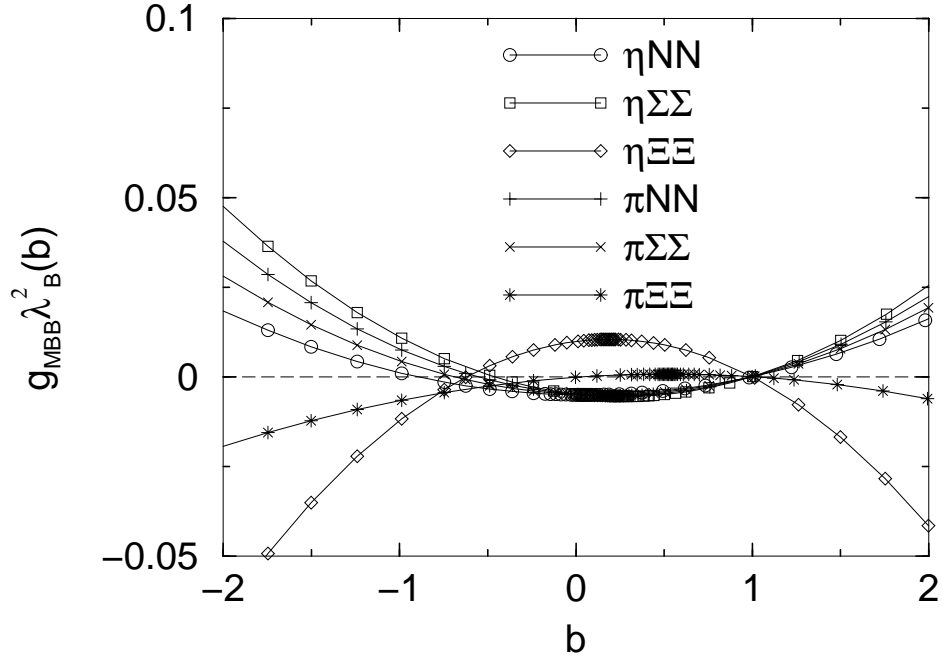


Figure 4:

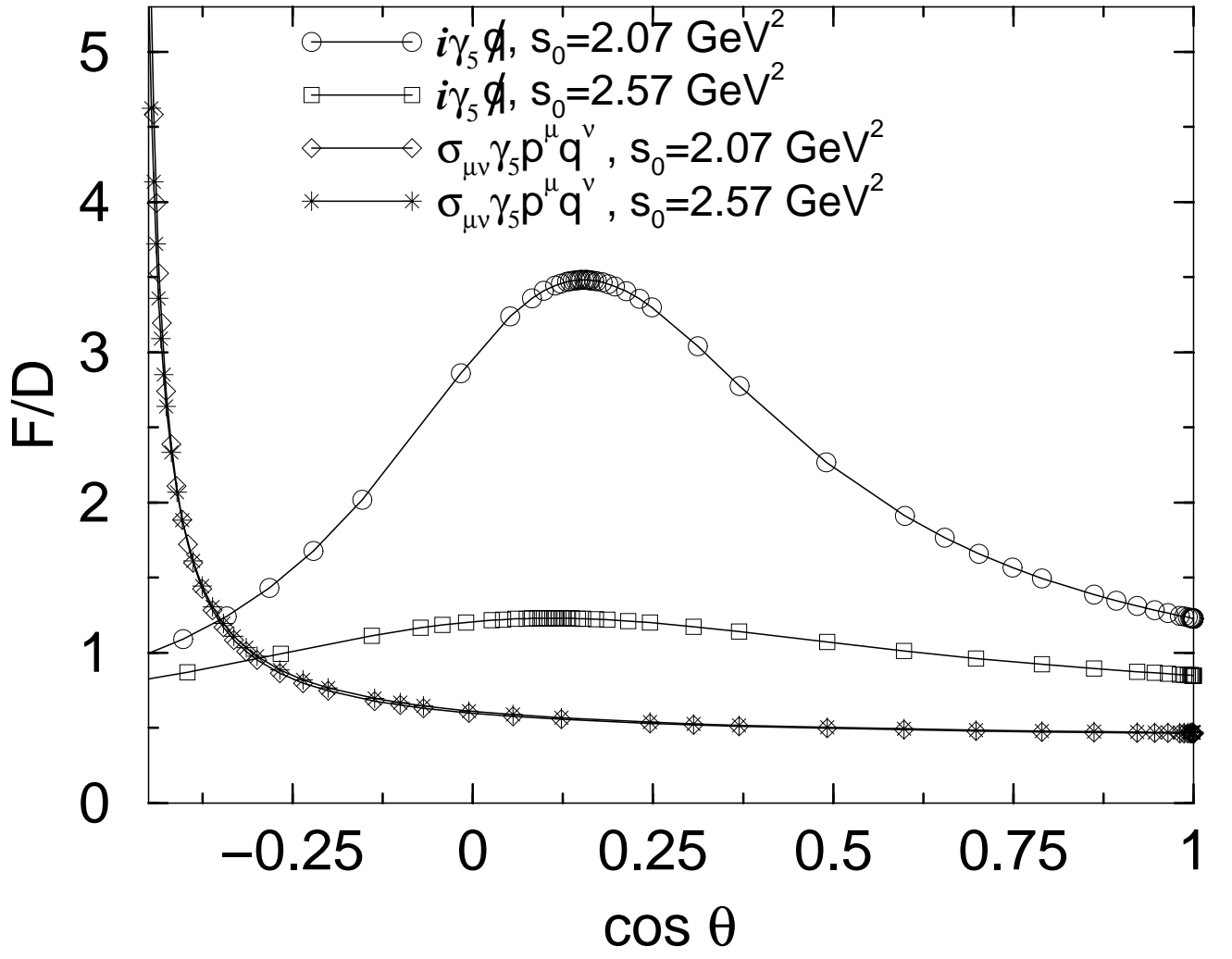


Figure 5: